Question 2α

Setup as in Q2 with $A = \begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix}$, $v_1 = (1,2), v_2 = (1,1)$.
Write Av_1 and Av_2 in the basis v_1, v_2.

$$Av_1 = \begin{pmatrix} 6 \\ 7 \end{pmatrix} = v_1 + 5v_2 \qquad Av_2 = \begin{pmatrix} 4 \\ 5 \end{pmatrix} = v_1 + 3v_2$$

The columns of A' are the coordinates of Av_1 and Av_2 in the basis v_1, v_2 and the columns of P are v_1 and v_2. Hence

$$A' = \begin{pmatrix} 1 & 1 \\ 5 & 3 \end{pmatrix}, \qquad P = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}.$$

Question 3

Very similar to example above.

Question 4

A polynomial of degree ≤ 3 can be written as $aX^3 + bX^2 + cX + d$. Use this along with Lemma 3 from notes.

Question 5

First find the null space (similar to end of sheet 2). This should give you the two vectors that form a basis for N_A at least. Try to find the simplest vectors you can add which will give you a basis for $\mathbb{R}^4$.

Question 7

When working with linear dependence over $\mathbb{C}$, write the coefficients as, e.g. $c = a+ib$, then consider real and imaginary parts separately.