MA10210: ALGEBRA 1B

http://people.bath.ac.uk/aik22/ma10210

Comments on Sheet 4

- Explanations need to be more thorough

Good writing is clearly important if you wish to be understood, but it has a bonus: it clarifies for you the material being communicated and thus adds to your understanding. In fact, I believe that if I can't explain an idea in writing, then I don't understand it. This is one reason why writing well helps you to think like a mathematician.

Generally, we write to explain to another person, so have this person in mind. Two points to remember:

- Have mercy on the reader. Do not make it difficult for them – particularly someone marking your work.
- The responsibility of communication lies with you. If someone at your level can't understand it, then the problem is with your writing!

Kevin Houston, *How to Think Like a Mathematician,* *sample chapter*

Comments on Sheet 4

- Working in different bases:
  - Decide on the basis you are using – make it clear
  - When writing a matrix in your basis, the answer should be given in the basis you are using.
  - Consider Q4:

$$\phi(p_1) = 1 = p_0 \Rightarrow A \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\phi(p_2) = 2x - 1 = p_0 + 2p_1 \Rightarrow A \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

    - Repeat this process for each of the basis elements

Warm-up Questions

- Q1
- Q4
- Q6

- Bonus Question:
  - Which combinations of the following five elements form a basis of $\mathbb{R}^4$?

$$\begin{pmatrix}0 & 1 & 0 & 1\end{pmatrix} \quad \begin{pmatrix}0 & 0 & 0 & 1\end{pmatrix} \quad \begin{pmatrix}0 & 1 & 0 & 0\end{pmatrix}$$

$$\begin{pmatrix}1 & 2 & 4 & 3\end{pmatrix} \quad \begin{pmatrix}0 & 3 & 3 & 0\end{pmatrix}$$

Answers to bonus question

- A B C D E

$$(0\quad 1\quad 0\quad 1)\ (0\quad 0\quad 0\quad 1)\ (0\quad 1\quad 0\quad 0)\ (1\quad 2\quad 4\quad 3)\ (0\quad 3\quad 3\quad 0)$$

- Try creating the standard basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ using A-E
  - Missing A:

    $\mathbf{e}_1$ = -3B-2/3C+D-4/3E;

    $\mathbf{e}_2$ = C; $\mathbf{e}_3$ = -C+1/3E; $\mathbf{e}_4$ = B;

    So {B, C, D, E} is a basis for $\mathbb{R}^4$.
  - Missing B: {A, C, D, E} is a basis (note that B=A-C)
  - Missing C: {A, B, D, E} is a basis (C=A-B)
  - Missing D:

    no combination of {A, B, C, E} will give $\mathbf{e}_1$, so not a basis.
  - Missing E: Not a basis. (same reason as for missing D)

Overview of Sheet 5

- Q2: similar to Q1
- Q3: Use Theorem 3.1.4, Prop 3.2.3,
  Theorem 3.2.5 (for part iii),
  Lemma 3.2.1 (for part iv)
- Q5: (i) use Q4; (ii) use similar process to Q4
- Q7: similar to Q6

Overview of Sheet 5

- Q8:
  - (ii) Assume there is some dependence and try various values of **x** to see what the coefficients need to be.
  - (iii) Follow instructions!
  - (iv) Consider how (i)-(iii) works and apply the same method to the case in (iv) (which has four dimensions – why?)