MA10209 – Week 9 Tutorial

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Top Tips (response to sheet 8)

- The best way to show a polynomial is reducible is to write it as a product of its factors.
- "If and only if" requires two directions.
- If you only need to prove one direction, make sure you prove the right one (e.g. 7a)

Working with matrices

$$\text{Let } A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- Give a geometric interpretation of A and B.
- Verify that $A^4 = B^2 = I$, and that $BA = A^3B$.
- Show that the smallest group that contains A and B has eight elements.
- Find the smallest non-negative integers m, n such that $A^m B^n = A^2 B^5 A^3 B^3 A B^2 A B A$.

Cycle notation

- Give the following in cycle notation:

α	β	γ
$1 \mapsto 3$	$1 \mapsto 1$	$1 \mapsto 4$
$2 \mapsto 5$	$2 \mapsto 3$	$2 \mapsto 3$
$3 \mapsto 1$	$3 \mapsto 5$	$3 \mapsto 2$
$4 \mapsto 2$	$4 \mapsto 2$	$4 \mapsto 1$
$5 \mapsto 4$	$5 \mapsto 4$	$5 \mapsto 5$

- Write the following in the form used above:

$$\delta = (2\ 5) \qquad \varepsilon = (1\ 3\ 5)(2\ 4)$$

Cycle notation

- Give the following in cycle notation:

α	β	γ
$1 \mapsto 3$	$1 \mapsto 1$	$1 \mapsto 4$
$2 \mapsto 5$	$2 \mapsto 3$	$2 \mapsto 3$
$3 \mapsto 1$	$3 \mapsto 5$	$3 \mapsto 2$
$4 \mapsto 2$	$4 \mapsto 2$	$4 \mapsto 1$
$5 \mapsto 4$	$5 \mapsto 4$	$5 \mapsto 5$
$\alpha = (1\ 3)(2\ 5\ 4)$	$\beta = (2\ 3\ 5\ 4)$	$\gamma = (1\ 4)(2\ 3)$

Cycle notation

- Write the following in the form used above:

$\delta = (2\ 5)$ $\qquad$ $\varepsilon = (1\ 3\ 5)(2\ 4)$

δ	ε
$1 \mapsto 1$	$1 \mapsto 3$
$2 \mapsto 5$	$2 \mapsto 4$
$3 \mapsto 3$	$3 \mapsto 5$
$4 \mapsto 4$	$4 \mapsto 2$
$5 \mapsto 2$	$5 \mapsto 1$

Cycle notation

- Give the following in cycle notation:

$$(2\ 3) \circ (3\ 4\ 5)$$

$$(3\ 4\ 5) \circ (2\ 3)$$

$$(1\ 2) \circ (1\ 3\ 2\ 4) \circ (1\ 2)$$

Cycle notation

- Give the following in cycle notation:

$$(2\ 3) \circ (3\ 4\ 5)$$

$$1 \mapsto 1 \mapsto 1$$
$$3 \mapsto 2 \mapsto 2$$
$$4 \mapsto 4 \mapsto 3$$
$$5 \mapsto 5 \mapsto 4$$
$$2 \mapsto 3 \mapsto 5$$

$$= (2\ 3\ 4\ 5)$$

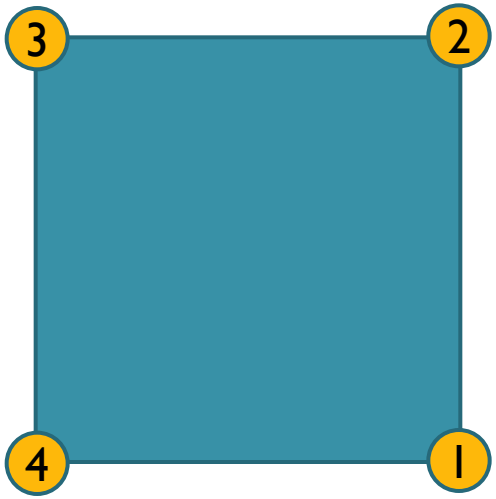
$$(3\ 4\ 5) \circ (2\ 3)$$

$$1 \mapsto 1 \mapsto 1$$
$$4 \mapsto 3 \mapsto 2$$
$$2 \mapsto 2 \mapsto 3$$
$$5 \mapsto 4 \mapsto 4$$
$$3 \mapsto 5 \mapsto 5$$

$$= (2\ 4\ 5\ 3)$$

D_8 and cycle notation

- D_8 is the group of symmetries of the square:



$$a = (1\ 2\ 3\ 4)$$
$$b = (1\ 2)(3\ 4)$$

Give a geometric interpretation of a and b.

Verify that $a^4 = b^2 = 1$, and that $ba = a^3b$.

Exercise sheet 9 - Overview

- Q1 – similar to example
- Q2 – techniques used at the start of Sheet 5 are relevant (integral entries means entries are integers)
- Q3
  - (a) – similar to sudoku
- Q4 – same as Q7 on Sheet 8 but with different notation
- Q5
  - (a) look up angle formulas
  - (b) use (a)

Exercise sheet 9 - Overview

- Q6 – neatest option is to use cycle notation to represent the bijections
- Q8 – see Q3(c)
- Q9 – two directions: one is quick, the other can be completed using contradiction