MA10209 – Week 8 Tutorial

B3/B4, Andrew Kennedy

Top Tips (response to sheet 7)

- Be careful with calculations, this is where a lot of the mistakes occurred
- Check answers when you can
- Make sure your answer is clear and undisputable
- Keep it short – answer the problem rather than telling me the story

Exercise Sheet 7 - Overview

- Q1 – similar to tutorial example
- Q2 – 9 divides 2n+1 $\implies$ 2n+1 = 0 mod 9
  $\implies$ 2n = -1 mod 9
- Q3 – don't unpick Euclid if you can avoid it – try to spot a number that satisfies the conditions
- Q4 – we want to find a number *u* such that

  $n_i | u + i - 1$ for $i = 1, 2, 3, \ldots, 1000$
  where n_i is the product of 1000 prime numbers for each i.

- Q5 – *u*, *d* coprime, so there exist λ, μ such that

  $1 = \lambda u + \mu d.$

Polynomials

- Consider $x^2 + 9$
- Is this irreducible in $\mathbb{Q}[X]$? (see Q3)
- Is this irreducible in $\mathbb{R}[X]$?
- Is this irreducible in $\mathbb{C}[X]$?

Remainder Theorem

- Let $p(x) = (x - a)q(x) + r(x)$ with $\deg r < 1$
  Then $r = p(a)$

Remainder Theorem

- Consider the following polynomial:

$$x^4 + x^3 + 14x^2 + 16x - 32$$

- Use the remainder theorem to find which of the following are roots:
  - 1 -1 2 -2
- Factorise the polynomial into irreducible factors, considering it as a polynomial in
  - $\mathbb{R}[X]$
  - $\mathbb{C}[X]$

Exercise Sheet 8 - Overview

- Q1 – definition is given in the question, so work from there
- Q2 – use long polynomial division (unless you can spot another method)
- Q3 – irreducible polynomials
- Q4 – If it helps, think of a+bi
- Q5
  - (a) FTA gives that we can find a root α
  - (b) roots come in conjugate pairs
  - (c) use induction

Exercise sheet 8 - Overview

- Q6 – need to do two directions. As is common with such things, one way is easier than the other.
  (You can get away with only considering leading terms if you do it right).
- Q7 – New question
  - good practice with groups
- Q8 – Harder than Q7 since you can't really put your hands on the matrices
  - requires explanation rather than calculation
- Q9 – 'Playing' with matrices
  - matrix multiplication in summation form